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О ВЫВОДЕ ОБОБЩЕННОЙ ГРАВИТАЦИОННОЙ ЭНТРОПИИ

Аннотация. Представлен новый вывод обобщенной гравитационной энтропии, связанной с поверхностями «перепутывания» коразмерности 2. Предлагаемый подход близок к «гамильтонову» методу Джакобсона-Майерса, в том смысле, что энтропия возникает из граничного слагаемого в гравитационном действии, когда выделяется малая область вблизи поверхности перепутывания. В наших аргументах мы используем идею Мальдасены-Левковича и интерпретируем граничное слагаемое а гравитационном действии как действие "космической струны" (браны). Однако важное отличие нашего подхода от первоначальной формулировки обобщенной гравитационной энтропии Мальдасены и Левковича в том, что мы не используем многообразия с коническими сингулярностями как инструмент проведения расчетов. Вариации гравитационных действий по параметру реплик подразумевают изменение положения "космической струны". Требуя, что поверхность перепутывания является экстремумом функционала энтропии, мы приходим к формуле, которая совпадает с известным результатом для энтропии черной дыры, когда поверхность перепутывания отождествляется с горизонтом. В применении нашего подхода к теориям гравитации в форме Лавлока формула для обобщенной энтропии совпадает с результатами, полученными другими методами.
Ключевые слова: энтропия квантового перепутывания, теории гравитации с высшими производными, квантовая гравитация.

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NOTES ON DERIVATION OF GENERALIZED GRAVITATIONAL ENTROPY

Abstract. A novel derivation of generalized gravitational entropy associated to co-dimension 2 'entangling' hypersurfaces is given. The approach is similar to the Jacobson- Myers 'Hamiltonian' method in a sense that the entropy appears from a boundary term in the action when one isolates a small domain around the entangling surface. In our arguments we also use the idea by Lewkowycz and Maldacena and interpret the boundary term in the gravity action as a 'cosmic string' (brane) action. However, the important difference between our approach and the original formulation of the generalized gravitational entropy by Lewkowycz and Maldacena is that we never use manifolds with conical singularities as a tool to carry out the computations. Variations of gravity actions over the replica parameter imply changing position of the 'cosmic string'. By requiring that the entangling surface is an extremum of the entropy functional we come to the entropy formula which coincides with known results for black hole entropy formula when the entangling surface is a black hole horizon.When our approach is applied to Lovelock theories of gravity the generalized entropy formula coincides with results derived by other methods.

Key words: entropy of quantum entanglement, higher derivative gravity theories, quantum gravity

1. Introduction

There is a mounting number of arguments that the Bekenstein-Hawking entropy can be applied not only in case of black hole horizons but to arbitrary co-dimension 2 surfaces in flat and curved spacetimes. First arguments that this can be done in a consistent way have been presented in the work of the present author [7; 8]. If B is a minimal hypersurface in a constant time slice Σ of a stationary spacetime M which is a solution to the Einstein theory one can associate to this surface an entropy [8]

$$S(B) = \frac{A(B)}{4G}. \qquad (1.1)$$

Equation (1.1) has been inspired by the holographic formula [14] for computing entanglement entropy in conformal theories with gravity duals. *S(B)* can be interpreted as an entanglement entropy in quantum gravity [8]. A similar concept of spacetime entanglement was discussed in a number of publications, see e.g. [1; 12].

Recently formula (1.1) has been also proposed by Lewkowycz and Maldacena [11] as a 'generalized gravitational entropy'. The authors of [11] considered a general setup when M is an arbitrary (not necessarily stationary) solution of the Einstein gravity. It was assumed that boundary ∂M of M has non-contractable circles S^1 which are contractable inside M on B. When B is minimal in M equation (1.1) yields an entropy associated to a density matrix specified by the given boundary conditions. It was argued that the above construction also has an entanglement interpretation.

The Maldacena-Lewkowycz proposal and its extensions to higher derivative gravities attracted a considerable interest [2–6]. The main difficulty here was related to a careful treatment of conical singularities in gravity actions [9]. The singularities appeared in [11] at some steps of computations.

The aim of the present work is to derive the generalized gravitational entropy without any use of conical singularities. Our approach is similar to the

Jaconson-Myers 'Hamiltonian' method [10] in a sense that the entropy appears from a boundary term in the action when one isolates a small domain around the entangling surface B. We prove the extremality of the entropy functional on the entangling surface and test our approach in Lovelock theories of gravity.

After necessary definitions in Sec. 2 the suggested method is introduced in Sec. 3. Applications to higher derivative gravities are considered in Sec. 4 followed by a brief discussion in Sec. 5.

2. Definitions

Entanglement entropy in a quantum gravity, as suggested in [8], is specified by the boundary conditions, which imply a holographic nature of the theory. One starts with a class of manifolds M with the boundary condition $\partial M=T$, where T is a $d-1$ dimensional manifold. The entanglement entropy of [7; 8] and the generalized gravitational entropy of [11] can be defined in terms of an 'entanglement' partition function $Z[T_n]$ where n=1,2,…, and T_n are boundary manifolds constructed from n copies of T. Construction of T_n is similar to a construction of 'replicated' manifolds in a QFT to represent quantities like Tr ρ^n, where ρ is a reduced density matrix obtained by tracing over unobservable states.

The entanglement partition function $Z[T_n]$ is defined by quantum gravity theory, where bulk geometries M_n have the boundary $\partial M_n = T_n$. One may represent $Z[T_n]$ in terms of some integral over 'histories' with above boundary conditions and integration measure defined by some low-energy action $I[M_n]$. In a semiclassical approximation $\ln Z[T_n] \cong -I[\overline{M}_n]$, where $\overline{M}_n$ realizes a minimum of the action for given boundary conditions, and the entropy can be defined as [8]

$$S = \lim_{n\to 1}\left(n\partial_n - 1\right) I[\overline{M}_n]\cdot \qquad (2.1)$$

One first finds the action for integer n, assumes that n can be replaced with a continuous parameter, and then goes to n=1. This is a common trick used in statistical physics known as a replica method.

The Maldacena-Lewkowycz approach is to look for $\overline{M}_n$ as regular solutions to the corresponding low-energy gravity equations with the condition $\partial\overline{M}_n = T_n$. We assume that $\overline{M} = \overline{M}_1$ is one of solutions for standard boundary conditions $\partial\overline{M} = T$.

In [11] the boundary manifolds T are required to have non-contractable circles S^1. One can introduce a coordinate τ along the circles with the period 2π. The boundary manifold T_n for the partition function in the replica method is glued smoothly from n copies of T such that τ has the period $2\pi n$. It is required that T and T_n are boundaries of manifolds where S^1 can be contracted in the bulk. A simple example is the case of a black hole instanton, where M is a solid hypertorus for $\partial M= S^1\times S^{d-1}$.

In the rest of the paper we use the following notations: $R_{\mu\upsilon\lambda\rho}$ is the Riemann tensor of a d dimensional manifold M, M has the Euclidean signature. The Greek indexes run from 1 to d. ∂M is a boundary of M, K_b^a is the extrinsic curvature tensor of ∂M. The Latin indexes a, b, c, d run from 1 to d–1. A tensor R_{abcd} on ∂M is a projection of the Riemann tensor of M on a space tangent to ∂M. B is an 'entangling' co-dimension 2 hypersurface in M. We use a unit complex vector constructed from two normal vectors to B and define the corresponding complex extrinsic curvature k_j^i. The Riemann tensor defined by the metric of B is denoted as $\hat{R}_{ijkl}$. The Latin indexes i, j, k, l run from 1 to d–2. The operation $[\mu_1,\ldots,\mu_p]$denotes totally antisymmetric combination of p p indexes (accompanied by the factor $1/p!$).

3. A novel derivation of the generalized entropy

To present the method we start with the Einstein gravity. Let $\overline{M}_n$ be a solution to gravity equations for corresponding boundary conditions T_n. Let B_n be an extremal surface in $\overline{M}_n$ where S^1 are contracted. Maldacena and Lewkowycz [11] interpret B_n as a world-sheet of a cosmic string (brane) and derive conditions on B_n from a regularity condition on the geometry around a cosmic string. We consider sets of solutions $\overline{M}_n$ and corresponding surfaces B_n but do not write the index n explicitly for a while.

The 'cosmic string' action on B_n can be inferred immediately from the gravity action on $\overline{M}$. The idea is the following. Consider a small neighbourhood N_ε around B, where the metric behaves as [9]

$$ds^2 \cong r^2 d\tau^2 + dr^2 + \left(\gamma_{ij}(\upsilon) + 2r\left(\cos\ \tau k_{ij}^{(1)}(\upsilon) + \sin\ \tau k_{ij}^{(2)}(\upsilon)\right)\right)^2 d\upsilon^i d\upsilon^j \quad (3.2)$$

Here $0 \le \tau < 2\pi n$, $\gamma_{ij}(\upsilon)$ is a metric on B, and $k_{ij}^{(p)}$ are two extrinsic curvatures of B. In coordinates (3.2) the boundary of the neighbourhood is chosen to be located at $r=\varepsilon$.

The gravity action on $\overline{M}$ is decomposed on the action on N_ε and the action on $\overline{M}/N_\varepsilon$. It is assumed that a necessary boundary term with the extrinsic curvature on the boundary of N_ε is included in the actions to have a well-posed variational problem. In the limit $\varepsilon \to 0$ the action on N_ε can be interpreted as a 'cosmic string' action I_{str},

$$I_{\text{str}}[B] = \lim_{\varepsilon\to 0} I[N_\varepsilon] = -A(B)/(4G), \quad (3.3)$$

$$I[N_\varepsilon] = -\frac{1}{16\pi G}\int_{N_\varepsilon}\sqrt{g}\,d^d xR - \frac{1}{8\pi G}\int_{\partial N_\varepsilon}\sqrt{h}\,d^{d-1}yK\ , \quad (3.4)$$

To get (3.3) from (3.4) one should take into account that the extrinsic curvature tensor K^{μ}_{υ} of N_ε has a singular component $K^{\tau}_{\tau}=1/\varepsilon$. This singularity is compensated by the factor ε in the integration measure. The bulk part of $I[N_\varepsilon]$vanishes in this limit since N_ε is a regular manifold. Note that a 'cosmic string' has the negative tension $-1/(4G)$. Thus, the use of this terminology is only for an analogy, not for drawing any physical consequences.

In the limit $\varepsilon\to 0$ one can write

$$I[\overline{M}]=I[\overline{M}^c]+I_{str}[B], \qquad (3.5)$$

where $I[\overline{M}^c]$ is an action on a manifold $\overline{M}^c=\overline{M}/B$, where B is removed. Variation of (3.5) over the metric yields the Einstein equations outside B. These are the vacuum equations if the matter is absent.

Variation of the 'string action' is easy to understand at a small but finite ε (at a finite brane thickness). There are non-trivial variations on the boundary N_ε due to the boundary terms in the gravity action on $\overline{M}/N_\varepsilon$ and in the 'string' domain N_ε. This yields equations

$$\left(K^{\mu\upsilon}-h^{\mu\upsilon}K\right)_+=-\left(K^{\mu\upsilon}-h^{\mu\upsilon}K\right)_-\equiv 8\pi G t^{\mu\upsilon}_{str}\,. \qquad (3.6)$$

Here $(K_+)^{\mu}_{\upsilon}$ and $(K_-)^{\mu}_{\upsilon}$ are the extrinsic curvatures of N_ε in $\overline{M}/N_\varepsilon$ and N_ε, respectively. The left hand side comes out from the 'gravity part' and the right hand side from the 'string'. The r.h.s. of (3.6) is interpreted as a stress-energy tensor of the 'string'. Equations (3.6) are identities since the division on the gravity and 'string' parts is artificial.

From now on the index n is restored. Before applying formula (2.1) we discuss variation of $I[\overline{M}_n]$ over n. We use the same arguments as in [11] and consider $I[\overline{M}_n]$ as some integrals at continuous n.

Let us start with decomposition (3.5). For $I[\overline{M}^c_n]$ extrapolation to continuous n does not pose a problem. There appear no conical singularities on $\overline{M}^c_n$ since a small domain near B_n is excluded. Variation over n can be written as

$$\partial I[\overline{M}^c_n]=\partial^{\text{int}}_n I[\overline{M}^c_n]+\partial^{\boldsymbol{bulk}}_n I[\overline{M}^c_n]+\partial^{\boldsymbol{boun}}_n I[\overline{M}^c_n]\,. \qquad (3.7)$$

The operation ∂^{int}_n means a change of the upper limit in the integrals in τ, when the integrand itself is fixed. Variations $\partial^{\text{bulk}}_n,\partial^{\text{boun}}_n$ take into account, respectively, change of metrics in the bulk and on the boundaries of $\overline{M}^c_n$. Variation of the string action can be written as

$$\partial_n I_{str}[B_n]=\partial^{\boldsymbol{metr}}_n I_{str}[B_n]+\partial^{\boldsymbol{pos}}_n I_{str}[B_n], \qquad (3.8)$$

where ∂^{metr}_n corresponds to the variation of the metric of B_n, while ∂^{pos}_n takes into account change in the position of B_n under fixed metric. If B_n is a minimal surface the change of the position does not change the string action in the leading order.

We need variations at n=1. Since the operation $\partial_n^{\rm int}$ (3.7) does not change the metric one has

$$\lim_{n\to 1}\partial_n^{\rm int} I[\overline{M}_n^c] = I[\overline{M}^c], \qquad (3.9)$$

where $\overline{M}^c = \overline{M}_1^c$. Equation (3.9) is easy to understand when the metric does not depend on τ. In general case one should consider changing the integration limits as changing the number of integrals $I[\overline{M}^c]$. One also has

$$\lim_{n\to 1}\partial_n^{\boldsymbol{bulk}} I[\overline{M}_n^c] = 0\,. \qquad (3.10)$$

The action has an extremum on $\overline{M}_n^c$.

Since the metric on the external boundary is fixed one should care about variations on the internal boundary of $\overline{M}_n^c$. The latter are compensated by the variations of the string action,

$$\partial_n^{\boldsymbol{metr}} I_{\boldsymbol{str}}[B_n] + \partial_n^{\boldsymbol{boun}} I[\overline{M}_n^c] = 0\,. \qquad (3.11)$$

Eq. (3.11) is ensured by gravity equations (3.6) in the presence of the string.

By taking into account equations (3.8), (3.9), (3.10), (3.11) one finds

$$\lim_{n\to 1}\partial_n I[\overline{M}_n] = I[\overline{M}] + \lim_{n\to 1}\partial_n^{\boldsymbol{pos}} I_{\boldsymbol{str}}[B_n]\,. \qquad (3.12)$$

$$S = -I_{\boldsymbol{str}}[B_n] + \lim_{n\to 1}\partial_n^{\boldsymbol{pos}} I_{\boldsymbol{str}}[B_n]\,. \qquad (3.13)$$

The Bekenstein-Hawking formula (1.1) follows from (3.13) if one uses (3.3) and assumes that B_n is a minimal surface.

4. Entropy formula in the Lovelock gravity

From Eqs. (3.3), (3.13) the generalized entropy can be written as

$$S = -I_{\boldsymbol{str}}[B] = -\lim_{\varepsilon\to 0} I[N_\varepsilon]\,. \qquad (4.14)$$

It is important that this equality does not require that the theory is of the Einstein form. It can be applied to other theories, e.g. higher derivative gravities provided that the action functional $I[N_\varepsilon]$ includes boundary terms which insure well-posed variational procedure. (Normal derivatives of the metric variations should not appear on the boundary.) Another requirement is that B is an extremum of $I_{\rm str}[B]$. Remember that this condition eliminates the last term in the r.h.s. of (3.13).

An example of a higher derivative gravity, where the required boundary terms are known, is the Lovelock theory

$$I_L[M] = -\sum_m c_m\left(\int_M \sqrt{g}\,d^d x L_m + \int_{\partial M}\sqrt{h}\,d^{d-1}y D_m\right). \qquad (4.15)$$

Here c_m are some coefficients, c_1>0, and

$$L_m = \frac{(2m)!}{2^m} R^{[\mu_1\nu_1}_{[\mu_1\nu_1} R^{\mu_2\nu_2}_{\mu_2\nu_2} \dots R^{\mu_m\nu_m]}_{\mu_m\nu_m]}, \qquad (4.16)$$

$$D_m = \frac{(2m)!}{2^{m-1}} \sum_{p=0}^{m-1} d_{m,p} K^{[a_1}_{[a_1} K^{a_2}_{a_2} \dots K^{a_{2p+1}}_{a_{2p+1}} R^{b_1c_1}_{b_1c_1} R^{b_2c_2}_{b_2c_2} \dots R^{b_{m-p-1}c_{m-p-1}]}_{b_{m-p-1}c_{m-p-1}]}, \qquad (4.17)$$

$$d_{m,p} = \frac{(m-1)!2^{3p}\, p!}{(m-p-1)!(2p+1)!}. \qquad (4.18)$$

It is implied that curvatures in the r.h.s. of (4.17) are taken on B. We use the form of the boundary term (4.17) given in [13].

Consider the Lovelock action in a small domain N_ε where the metric behaves as in (3.2). As earlier, we place the boundary ∂N_ε at $r=\varepsilon$. The 'string action' in this theory is determined by the boundary terms on ∂N_ε.

We need to study boundary terms in (4.15) in the limit $\varepsilon\to 0$. Since the only singular component of K^a_b is $K^\tau_\tau = 1/\varepsilon$ one can easily see that $B_m \sim 1/\varepsilon$ at $\varepsilon\to 0$. The singular terms can be easily extracted from (4.17):

$$K^{[a_1}_{[a_1} \dots K^{a_{2p+1}}_{a_{2p+1}} R^{b_1c_1}_{b_1c_1} \dots R^{b_{m-p-1}c_{m-p-1}]}_{b_{m-p-1}c_{m-p-1}]} \cong \frac{2p+1}{2m-1} K^\tau_\tau K^{[i_1}_{[i_1} \dots K^{i_{2p}}_{i_{2p}} R^{j_1k_1}_{j_1k_1} \dots R^{j_{m-p-1}k_{m-p-1}]}_{j_{m-p-1}k_{m-p-1}]}. \qquad (4.19)$$

The factor $(2p)!/p!$ in the r.h.s. of (4.19) appears since a pair of upper and lower τ indexes take $2p+1$ positions, $2m-1$ in the denominator results from the normalization factor in the operator [...]. The indexes i, j, k enumerate components of the curvature tensors in the directions tangent to B.

It is convenient to introduce complex extrinsic curvatures of B

$$k_{ij} = \frac{1}{2}\left(k^{(1)}_{ij} - ik^{(2)}_{ij}\right), \quad \bar{k}_{ij} = k^*_{ij}. \qquad (4.20)$$

Integration over the τ coordinate can be easily done,

$$\int_0^{2\pi n} d\tau K^{[a_1}_{[a_1} \dots K^{a_{2p+1}}_{a_{2p+1}} R^{b_1c_1}_{b_1c_1} \dots R^{b_{m-p-1}c_{m-p-1}]}_{b_{m-p-1}c_{m-p-1}]} \cong$$
$$\frac{2\pi n}{\varepsilon} \frac{2p+1}{2m-1} \frac{(2p)!}{p!} k^{[i_1}_{[i_1} \dots k^{i_p}_{i_p} \bar{k}^{i_{p+1}}_{i_{p+1}} \dots \bar{k}^{i_{2p}}_{i_{2p}} R^{j_1k_1}_{j_1k_1} \dots R^{j_{m-p-1}k_{m-p-1}]}_{j_{m-p-1}k_{m-p-1}]}. \qquad (4.21)$$

The factor (2p)!/p! in the r.h.s. of (4.21) counts the number of ways when p k-curvatures (or $\bar{k}$ -curvatures) appear from $2p$ K-curvatures.

When (4.21) is used in the boundary term (see (4.17)) one comes to the action

$$\lim_{\varepsilon\to 0} I_L[N_\varepsilon] = -4\pi n \sum_m m c_m \hat{I}_m[B], \qquad (4.22)$$

$$\hat{I}_m[B] = \int_B \hat{L}_{m-1}, \qquad (4.23)$$

$$\hat{L}_{m-1} = \frac{(2(m-1))!}{2^{m-1}} \sum_{p=0}^{m-1} \frac{2^{3p}(m-1)!}{p!(m-p-1)!} k^{[i_1}_{[i_1} \dots k^{i_p}_{i_p} \bar{k}^{i_{p+1}}_{i_{p+1}} \dots \bar{k}^{i_{2p}}_{i_{2p}} R^{j_1k_1}_{j_1k_1} \dots R^{j_{m-p-1}k_{m-p-1}]}_{j_{m-p-1}k_{m-p-1}]}. \qquad (4.24)$$

One can now see that the last equation (4.24) is of Lovelock form on B,

$$\hat{L}_{m-1} = \frac{(2(m-1))!}{2^{m-1}} \hat{R}^{[i_1 j_1}_{[i_1 j_1} \hat{R}^{i_2 j_2}_{i_2 j_2} \dots \hat{R}^{i_{m-1} j_{m-1}]}_{i_{m-1} j_{m-1}]} . \qquad (4.25)$$

Eqs. (4.24) follow from (4.25) if one uses in (4.25) the Gauss-Codazzi equations on B

$$\hat{R}^{j_1 j_2}_{i_1 i_2} = R^{j_1 j_2}_{i_2 i_2} + 2\left(k^{j_1}_{i_1} \bar{k}^{j_2}_{i_2} + \bar{k}^{j_1}_{i_1} k^{j_2}_{i_2} - k^{j_1}_{i_2} \bar{k}^{j_2}_{i_1} - \bar{k}^{j_1}_{i_2} k^{j_2}_{i_1}\right). \qquad (4.26)$$

Factor $(m–1)!/(p!(m–p–1)!)$ yields a number of ways to pick up p $k\bar{k}$ -pairs. Multiplier 2^{3p} takes into account factor 2 in the r.h.s. of (4.26) and the fact that each Riemann curvature in (4.26) produces 4 $k\bar{k}$ -pairs.

We come to the following formula of the generalized entropy associated to the surface B:

$$S = 4\pi \sum_m m c_m \hat{I}_m [B] . \qquad (4.27)$$

In particular case of the Gauss-Bonnet gravity this entropy formula has been obtained by different methods: in [9] by using 'off-shell' conical singularity method and in [2; 5] from the requirement of regularity of the geometry around the 'cosmic string'. For general Lovelock gravity our result coincides with formula derived in [6].

5. Discussion

We presented a sketch of arguments which may support the Maldacena-Lewkowycz proposal [11] when the low-energy gravity action has higher derivatives. We have not yet emphasized but implied that this construction should be applicable to holographic entanglement entropy. In this case B is a holographic entangling surface and the background manifold M is a solution to an AdS gravity.

Our arguments (and, perhaps, other derivations of the generalized gravitational entropy) cannot be considered as a sort of mathematical proof. One should prove that regular gravity solutions for given boundary conditions for each value of the replica index n do exist. If this is the case then the Maldacena-Lewkowycz entropy can be derived 'on-shell' without any use of manifolds with conical singularities.

In contrast to [11] the approach of [8] can be called 'off-shell'. By the construction the bulk manifolds M_n in [8] are replicas of M_1 with conical singularities at B. Understanding how the two ways to the gravitational entropy, [8] and [11], compliment each other would be a helpful step to resolve the remaining issues.

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